Contracts, risk-sharing and incentives

- Complete and incomplete contracts
- Verifiable vs. non-verifiable clauses (results, effort, contribution):
  - Risk sharing and moral hazard

Contracts, risk-sharing and incentives

- Risk aversion
- Absolute or relative risk aversion
  - The Arrow-Pratt measure of absolute risk-aversion (ARA):-U”(c)/U’(c).
    - Exponential utility functions belong to constant absolute risk-aversion family.
  - The Arrow-Pratt measure of relative risk-aversion (RRA):-cU”(c)/U’(c).
    - Constant relative risk-aversion implies decreasing absolute risk-aversion.
  - Empirical studies often provide evidence of decreasing absolute risk-aversion.

Contracts, risk-sharing and incentives

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500+	66.0	8.1	9.4	23.7	28.2	4382	58.8	1093

The basic agency model with hidden actions

- Basic assumptions:
  - Verifiable production (results)
  - The effort provided by an agent is not verifiable – hidden action
  - Risk neutral principal
  - Risk averse agent
  - Sequence of moves:
    - The principal offers a contract
    - The agent rejects or accepts the contract.
    - If the agent accepts the contract. he or she provides effort (if rejection they go their separate ways)
    - A random event occurs that affects the result of the agent's effort
    - Bothe the principal and the agent observe the result
    - The principal pays the agent according to the contracted remuneration scheme

The basic agency model with hidden actions

- Basic assumptions …..:
  - Effort is costly for the agent, $C(e)=0.5ce^2$ (conflict of interest)
  - Utility depends on remuneration (which the agent likes) and effort (which the agent dislike), $U[W-C(e)]=-\exp\{-a[W-C(e)]\}$ (CARA-type).
  - The agent can always attain UO outside the contract in the market.
  - Production: $y=e+\varepsilon$, where $\varepsilon \sim N(0, \sigma^2)$.
  - Principal introduces a linear contract: $W=k+by$,
    - k=fixed pay regardless of performance, b=piece-rate on performance.

- Strategy for solving the model:
  - Principal knows that the agent is utility maximizing. so step 1: find the agent's expected utility and maximize this w.r.t. effort.
  - Contingent on this info and the fact that pay must be so that the agent is willing to work, find k and b which maximize the principal's profit.

The basic agency model with hidden actions

- Maximize agent's expected utility:
  - $W=k+by=k+b(e+\varepsilon)=k+be+b\varepsilon$
  - $EU=E\{-\exp[-a(W-C(e))]\}=E\{-\exp[-a(k+be-C(e)+b\varepsilon)]\}$

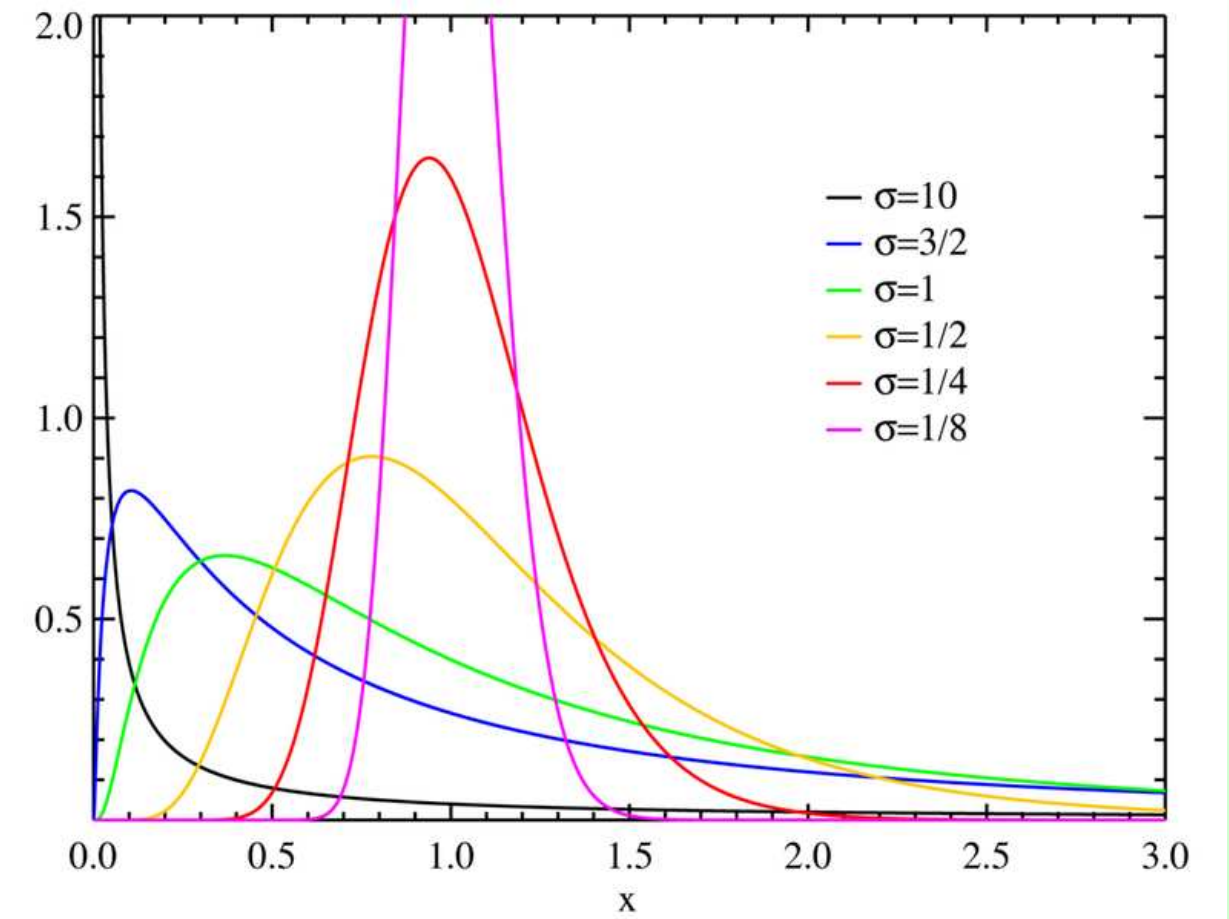
    $=E\{-\exp[-a(k+be-C(e))-ab\varepsilon)]\}$

    $=E\{-\exp[-a(k+be-C(e))]\exp[-ab\varepsilon]\}$

    $=-\exp[-a(k+be-C(e))]E\{\exp[-ab\varepsilon]\}.$

- Since $\varepsilon \sim N(0, \sigma^2)$ then $\exp[-ab\varepsilon] \sim$ Log Normal with mean$=\exp(0.5a^2b^2\sigma^2)$(see assignment/seminar)

The basic agency model with hidden actions



The basic agency model with hidden actions

- EU $=-\exp[-a(k+be-C(e))]\exp(0.5a^2b^2\sigma^2)$
  $=-\exp[-a(k+be-C(e)-0.5ab^2\sigma^2)$.

  - MAX EU w.r.t. e then gives: $C'(e^*)=b$ or $e^*=b/c$
    ($C(e)=0.5ce^2$).

  - Incentive-compatible constraint
  - Higher piece-rate on performance causes the agent to provide more effort.
  - The agent's effort provision does not depend on the fixed pay!

The basic agency model with hidden actions

- But the principal have to ensure that the agent accepts the contract. Remember the agent can always attain UO in the market.
- Thus: $UO \leq EU$, where $UO < 0$.
- Participation constraint
- Before we set up the principal's profit maximizing problem. recapitulate what do we know:
  - Agent's production: $y = e^* + \varepsilon$
  - Agent's remuneration: $W = k + b(e^* + \varepsilon)$
  - Principal's expected profit: $E\Pi = E(y-W) = Ey - EW = (1-b)e^* + k$ (remember $E\varepsilon = 0$)
  - Incentive constraint: $e^* = b/c$
  - Participation constraint: $UO \leq EU$

The basic agency model with hidden actions

- $MAX_{(k.b)} E\Pi = (1-b)e^{*}-k = e^{*}-be^{*}-k$ subject to: 1)$UO \leq EU$ (participation), 2)$e^{*}=b/c$ (incentive).

- Translate the participation constraint into monetary terms by introducing $x=-\ln(-UO)/a$
- Take the logarithm on both sides of the participation constraint: $x \leq k+be-C(e)-0.5ab^2\sigma^2$

- Since k does not affect e, the principal can adjust k so the participation constraint is satisfied for any optimum value of b.
- Rewrite the participation constraint: $x-be+C(e)+0.5ab^2\sigma^2 \leq k$

The basic agency model with hidden actions

- $MAX_b[e^*-C(e^*)-0.5ab^2\sigma^2-x]=[e^*-0.5ce^{*2}-0.5ab^2\sigma^2-x]$
  subject to: 1)$e^*=b/c$ (incentive).

- $MAX_b\,[b/c-0.5b^2/c-0.5ab^2\sigma^2-x]$
- $1/c-b/c-ab\sigma^2=0 \rightarrow b^*=1/[1+ac\sigma^2]$
- The piece-rate on performance should be set higher
  - the less risk averse the agent is,
  - the less costly effort is for the agent,
  - the less uncertain (or lower variance) the production is,

- What happens with the fixed pay?

The basic agency model with hidden actions

- There is no need to pay more than enough:

  $x-be+C(e)+0.5ab^2\sigma^2 = x-b(b/c) +0.5c(b/c)^2 + 0.5ab^2\sigma^2 = k$

- Rearrange, and correcting (unfortunate) mistake in C&Z, give:

  $k^*=x-0.5\{(1-ac\sigma^2)/c\}b^{*2}=x-0.5\{(1-ac\sigma^2)/c\}\{1/[1+ac\sigma^2]\}^2$

- The fixed pay should be set higher
  - the more risk averse the agent is,
  - the more costly effort is for the agent,
  - the more uncertain (or higher variance) the production is.

- The agent does not receive a wage premium in this case since the participation constraint is binding with equality!

The basic agency model withOUT hidden actions

- Assume effort is verifiable, what happens?

- The principal now determines e, b and k so the principal's expected profit is maximised. This contract can be defined as the *first-best* contract:

- MAX $_{(k.b.e)}$EΠ= e-be-k subject to: 1)UO≤EU
- MAX $_{(k.b.e)}$ [e-C(e) - $0.5ab^2\sigma^2$-x] $\rightarrow C'(e^o)=1 \rightarrow e^o=1/c$

  $\rightarrow b^o=0$

  $\rightarrow k^o=x-b^oe^o+C(e^o)+0.5ab^{o2}\sigma^2=x+C(e^o)$

- First-best contract completely insures the agent. fixed pay only!
- Effort and production under first-best contract higher than when effort not verifiable.

The basic agency model with hidden actions

- Assume agent is risk neutral but the agent's effort is unverifiable, what happens?

- Then a=0, which gives the following: $\rightarrow b^n=1$

  $\rightarrow k^n=x-0.5/c$

  $\rightarrow e^n=1/c.$

- Note that $e^n=1/c=e^o$. so production is equal in these two cases (first and second best contract coincide when agents are risk-neutral).

- When the agent is risk-averse and effort is unverifiable the fall in production is the price the principal has to pay to solve the problem of moral hazard.

An agency model with hidden actions and additional information

- Basic assumptions …..:
  - As before, but:
  - Introduce a signal, θ, correlated with the disturbance in the production ($y=e+\varepsilon$, where $\varepsilon \sim N(0, \sigma^2)$), $\theta \sim N(0, \sigma^2)$, $corr(\theta, \varepsilon)=\varrho\sigma^2$
  - The signal is verifiable!
  - Principal introduces a linear contract: $W=k+b_1y-b_2\theta$,

    k=fixed pay regardless of performance, b_1=piece-rate on own performance, b_2=piece-rate on additional information (signal).

- Strategy for solving the model:
  - Principal knows that the agent is utility maximizing, so step 1: find the agent's expected utility and maximize this w.r.t. effort.
  - Contingent on this info and the fact that pay must be so that the agent is willing to work, find k and b which maximize the principal's profit.

An agency model with hidden actions and additional information

- Expected utility of the agent (see assignment/seminar for derivation and details):

$$EU=-\exp\{-a[k+be-C(e)-0.5a\sigma^2(b_1^2+b_2^2-2\varrho b_1 b_2)]\}$$

  - MAX EU w.r.t. e then gives: $C'(e^*)= b_1$
  - Agent's production: $y= e^* + \varepsilon$
  - Agent's remuneration: $W=k+b_1(e^* + \varepsilon)-b_2\theta$
  - Principal's expected profit: $E\Pi=E(y-W)=Ey-EW=(1- b_1)e^* +k$
    (remember $E\varepsilon=0$, $E\theta =0$)
  - Incentive constraint: $C'(e^*)= b_1$
  - Participation constraint: $UO \leq EU$
    $x-be+C(e)+0.5a\sigma^2(b_1^2+b_2^2-2\varrho b_1 b_2) \leq k$

- Same technique as before, plug in part.-constr. and solve w.r.t $b_1$and b_2

An agency model with hidden actions and additional information

- MAX $_{b1.b2}[e^*-C(e^*)-0.5a\sigma^2(b_1^2+b_2^2-2\varrho b_1 b_2)-x]$ subject to:
  1) $C'(e^*)= b_1$ (incentive).

- Derivation (see seminar/assignment)
- Solution:→ $b_1^*=1/[1+ac\sigma^2(1-\varrho^2)]$ and $b_2^*= \varrho b_1^*$

  $k^*=x-0.5\{(1-ac\sigma^2(1-\varrho^2))/c\}\{1/[1+ac\sigma^2(1-\varrho^2)]\}^2$

- Implications:
  - As the signal provides more accurate information on the disturbance in production and thus the agent's effort, one increases the importance of the signal in the wage contract:
  - Extreme case 1: No information ($\varrho=0$), drop the signal from the contract
  - Extreme case 2: Complete information ($\varrho=1$), $W=k+y-\varrho\theta$

Contracts, risk-sharing and incentives 2003

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Why may performance pay be inefficient?

- Supervision and rent-seeking activities
- Multitasking
- The risk-profile of the firm's portfolio of projects
- Product market competition

Supervision and rentseeking

- Assumptions as the basic model. but:
  - Agent may invest in activity α so that a supervisior confirms that production is: $y'=y+\alpha$.
  - Agent receives remuneration: $W=k+b(y+\alpha)$
  - This impressing-supervisor-activity is however costly (in utility terms) for the agent: $F(\alpha)=0.5f\alpha^2$

- Agent's expected utility:$EU=-\exp[-a(k+b(e+\alpha)-C(e)-F(\alpha)-0.5ab^2\sigma^2)$.
  - MAX EU w.r.t. e and α then gives: $F'(\alpha)=C'(e^*)=b$, or $e^*=b/c$, $\alpha^*=b/f$.
  - Incentive constraints

Supervision and rentseeking

- The principal maximizes expected profit w.r.t. k and b, subject to the incentive constraints and the participation constraint. We derive in a similar way as before:
  - $b^*=1/[1+(c/f)+ac\sigma^2]$ and $e^*=b^*/c$, $\alpha^*=b^*/f$.
  - The possibility of rentseeking reduces the weight the principal puts on performance. if rent-seeking activities is cheap for the agent. then the principal is forced to provide a higher fixed pay and lower piece-rate.
  - Rent-seeking activities entail a loss of production value.
  - Rent-seeking activities make the first-best solution impossible to achieve even when agents are risk-neutral!

Why may performance pay be inefficient?

- Supervision and rent-seeking
- Multitasking
  - It is often desirable that an employee or a CEO diverts efforts to several tasks, not only one. If remuneration is based on verifiable factors, agents are provided with incentives to excell along these dimensions and not along those dimensions which are not incorporated in the remuneration scheme. Thus the achieved efforts may not corresponds to the optimal mix for the principal.
- The risk-profile of the firm's portfolio of projects
- Product market competition
- Solution to these problems: Stocks and stock options?

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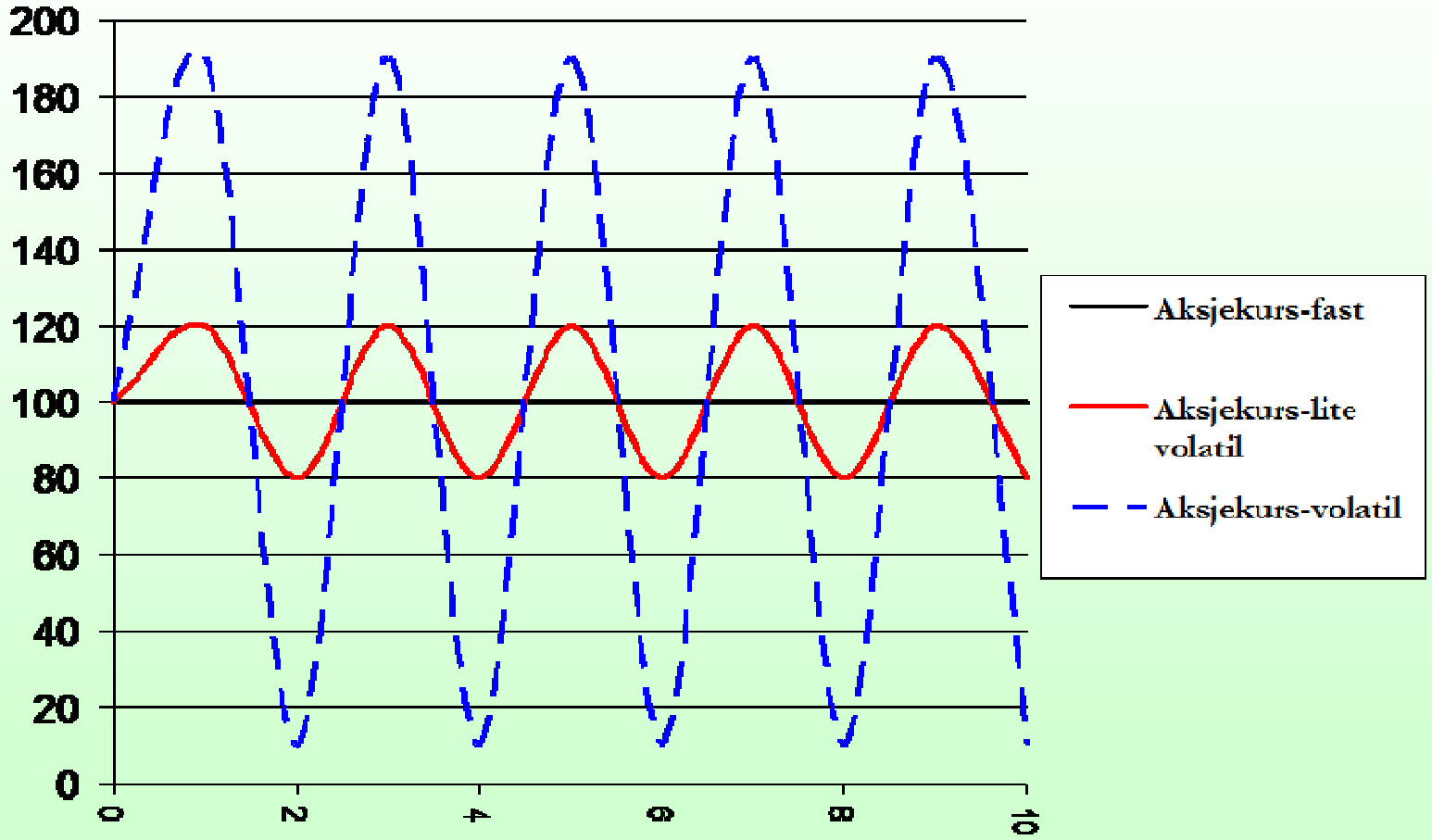
Stock options

- Stock option (”call”): The right to buy a stock at a price determined in advance on a later date or during a specified later period specified in advance.
- The value of the option depends on:
  - The underlying stockprice at the time of valuation (+).
  - The agreed upon price (”strike”) (-).
  - Time to exercise (+).
  - The asset’s (stock)volatility (+).
  - Risk free interest level (typically interest on government bonds)(+).
  - Restrictions on future possibilities on resale (-).

Stock options



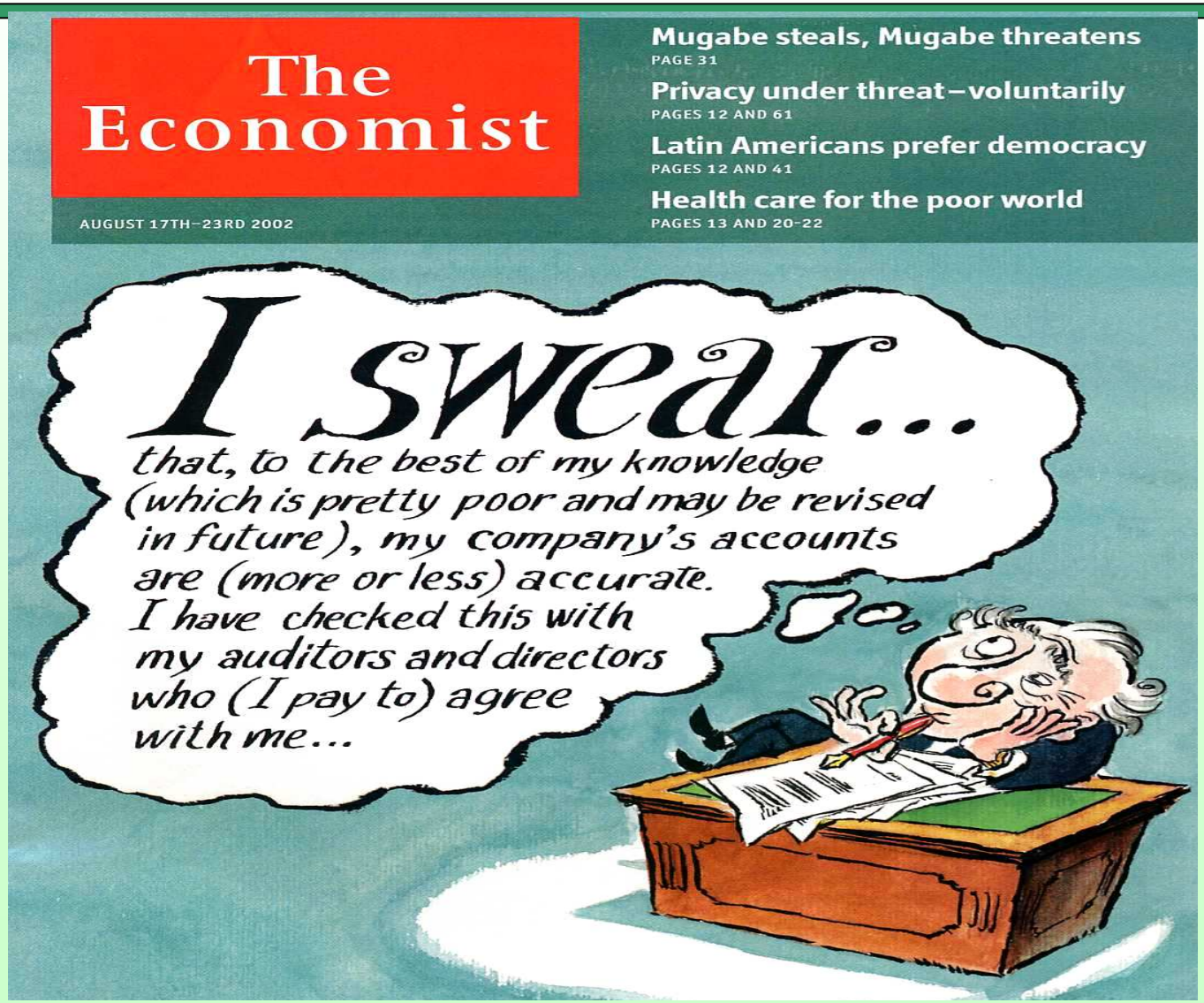
Stock options

Stockprice: 30 $	Degree of risk aversion								
10-years option	Normal			High			Very high		
Share in firm:	50	75	90	50	75	90	50	75	90
Strike 15 $	22.9	17.5	13.4	12.8	7.4	4.3	8.6	4.3	2.0
Strike 30 $	17.6	13.4	7.9	7.8	3.6	1.6	4.3	1.4	0.4
Strike 60 $	11.1	6.5	4.4	3.3	1.2	0.5	1.3	0.3	0.1

Stock options

- Signals about firm performance affect stock price, so these can be manipulated at least in the short-term,
- ”Unless executives posess an extraordinary ability to forecast future marketwide movement that drive these predicted returns, the results suggests that at least some of the awards are timed retroactively.”

Multitasking, rent-seeking and incentives



Stock options

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- ”Unless executives posess an extraordinary ability to forecast future marketwide movement that drive these predicted returns, the results suggests that at least some of the awards are timed retroactively.”
- The true costs associated with stock options are miscalculated (50% deviation when strikeprice is fair).
- For many firms the stock price is primarily determined by the world market!

Multitasking, rent-seeking and incentives